comparing managers. But the Sortino ratio should be compared only with other Sortino ratios and never with Sharpe ratios.

## Symmetric Downside-Risk Sharpe Ratio

The symmetric downside-risk (SDR) Sharpe ratio, which was introduced by William T. Ziemba,[6] is similar in intent and construction to the Sortino ratio, but makes a critical adjustment to remove the inherent upward bias in the Sortino ratio vis-à-vis the Sharpe ratio. The SDR Sharpe ratio is defined as the compounded return minus the risk-free return divided by the downside deviation. The downside deviation is calculated similarly to the downside deviation in the Sortino ratio with one critical exception: a multiplier of 2.0 is used to compensate for the fact that only returns below a specified benchmark contribute to the deviation calculation.[7] The benchmark used for calculating the downside deviation can be set to any level, but the same three choices listed for the MAR in the Sortino ratio would apply here as well: zero, risk-free return, and average return. (In his article, Ziemba uses zero as the benchmark value.) Unlike the Sortino ratio, the SDR Sharpe ratio (with the benchmark set to the average) can be directly compared with the Sharpe ratio.[8]

$$SDRSR = \frac{ACR - RF}{\sqrt{2} \times DD}$$

where $SDRSR$ = symmetric downside-risk Sharpe ratio
$ACR$ = annual compounded return
$RF$ = risk-free interest rate (e.g., T-bill return)
$DD$ = downside deviation

[6] William T. Ziemba, "The Symmetric Downside-Risk Sharpe Ratio," *Journal of Portfolio Management* (Fall 2005): 108–121.

[7] Ziemba used the term *benchmark* instead of MAR in defining downside deviation. If the median were used as the benchmark, only half the returns would be used to calculate the downside deviation, and a multiplier of 2.0 would then provide an exact compensating adjustment. For other choices for the benchmark (e.g., zero, risk-free return, average), the number of points below the benchmark would not necessarily be exactly half, and a multiplier of 2.0 would provide an approximate adjustment.

[8] To be perfectly precise, there would be a tendency for the SDR Sharpe ratio to be slightly lower for a symmetric distribution of returns because the SDR Sharpe ratio uses the compounded return rather than the arithmetic return used in the Sharpe ratio, and the arithmetic return will always be equal to or higher than the compounded return. If, however, zero or the risk-free return is used as the benchmark in the downside deviation calculation, assuming the manager's average return is greater than the risk-free return, there would be a tendency for the SDR Sharpe ratio to be higher than the Sharpe ratio for a symmetric distribution of returns for two reasons:

1. There will be fewer than half the returns below the benchmark, so the multiplication by 2.0 will not fully compensate.
2. Downside deviations from the risk-free return (and especially zero) would be smaller than deviations from the average.

These two factors would cause the downside deviation to be smaller than the standard deviation, implying a higher SDR Sharpe ratio than Sharpe ratio.

where *DD* is defined as:

$$DD = \sqrt{\frac{\sum_{i}^{N} (\min(X_i - \overline{X}, 0))^2}{N - 1}}$$

where $X_i$ = individual returns
$\overline{X}$ = benchmark return (e.g., mean, zero, risk-free)

Since the SDR Sharpe ratio includes only the downside deviation, multiplying by the square root of 2 (a consequence of doubling the squared deviations) is equivalent to assuming the upside deviation is equal (i.e., symmetric) to the downside deviation. This proxy replacement of the upside deviation is what makes it possible to compare SDR Sharpe ratio values with Sharpe ratio values.

The SDR Sharpe ratio (with any of the standard choices for a benchmark value) is preferable to the Sharpe ratio because it accounts for the very significant difference between the risk implications of downside deviations versus upside deviations as viewed from the perspective of the investor. The SDR Sharpe ratio is also preferable to the Sortino ratio because it is an almost identical calculation,[9] but with the important advantage of being directly comparable with the widely used Sharpe ratio. Also, by comparing a manager's SDR Sharpe ratio versus the Sharpe ratio, an investor can get a sense of whether the manager's returns are positively or negatively skewed.





## Gain-to-Pain Ratio

The gain-to-pain ratio (GPR) is the sum of all monthly returns divided by the absolute value of the sum of all monthly losses.[10] This performance measure indicates the ratio of cumulative net gain to the cumulative loss realized to achieve that gain. For example, a GPR of 1.0 would imply that, on average, an investor has to experience an amount of monthly losses equal to the net amount gained. The GPR penalizes all losses in proportion to their size, and upside volatility is beneficial since it impacts only the return portion of the ratio.

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[9] Besides the essential introduction of the 2.0 multiplier term, which allows unbiased comparisons between the SDR Sharpe ratio and the Sharpe ratio, the only difference between the SDR Sharpe ratio and the Sortino ratio is that it subtracts the risk-free return from the compounded return instead of the MAR (which may or may not be the risk-free return).

[10] The gain-to-pain ratio (GPR) is a performance statistic I have been using for many years. I am not aware of any prior use of this statistic, although the term is sometimes used as a generic reference for return/risk measures or a return/drawdown measure. The GPR is similar to the profit factor, which is a commonly used statistic in evaluating trading systems. The profit factor is defined as the sum of all profitable trades divided by the absolute value of the sum of all losing trades. The profit factor is applied to trades, whereas the GPR is applied to interval (e.g., monthly) returns. Algebraically, it can easily be shown that if the profit factor calculation were applied to monthly returns, the profit factor would equal GPR + 1 and would provide the same performance ordering as the GPR. For quantitatively oriented readers familiar with the omega function, note that the omega function evaluated at zero is also equal to GPR + 1.